MATH 1A − QUIZ 6 − SOLUTIONS

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(1) *(3 points)* Scientists recently discovered a drug called Peyamphetamine (Pa) which gives you a sudden boost of excitement as well as an eagerness to do math research! They discovered that when Pa is ingested in your body, the number of Pa−molecules grows at an exponential rate of 42 molecules/second. How long does it take until the number of Pa molecules in the body doubles?

We have $y' = 42y$ (in general, k is called the relative growth rate, or the exponential growth rate), so $y(t) = Ce^{42t}$.

Now we want to find t such that:

$$\begin{aligned} y(t) &= 2y(0) \\ Ce^{42t} &= 2C \\ e^{42t} &= 2 \\ 42t &= \ln(2) \\ t = \frac{\ln(2)}{42} \end{aligned}$$

(2) *(2 points)* Solve the differential equation $T'(t) = 3(T(t) - 2)$ with $T(0) = 6$.

If we let $y(t) = T(t) - 2$, then:

$y' = T' = 3(T - 2) = 3y$, so $y' = 3y$, so $y = Ce^{3t}$.

Now since $y = T - 2$, $C = y(0) = T(0) - 2 = 6 - 2 = 4$

Hence $y(t) = 4e^{3t}$, and so $\boxed{T(t) = y(t) + 2 = 4e^{3t} + 2}$

Other (similar) solution: Same as before, where we get: $y = Ce^{3t}$

But now $T = y + 2 = Ce^{3t} + 2$.

But $6 = T(0) = C + 2$, so $C = 4$, and hence $\boxed{T(t) = 4e^{3t} + 2}$

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(3) *(5 points)* Peyam's utility function U is given by the following **implicit** equation, where B is the happiness from eating broccoli sprouts and C is the happiness from eating cakes:

$$(U^2 - e^B)^3 = U^3 \ln(C)$$

Assume that at this very moment:

- Peyam's happiness from eating broccoli is $\ln(2)$ utils, and is decreasing by 2 utils/day
- Peyam's happiness from eating cake is e utils, and is increasing by $\frac{3e}{2}$ utils/day

Is Peyam getting happier or sadder at this moment, and at what rate?

Note: Assume that $U > 0$

Note: My apologies for those who were confused by the statement 'number of broccoli sprouts consumed'. What I meant to say was 'Happiness from eating broccoli sprouts'!

(1) No picture needed

(2) The equation is given: $(U^2 - e^B)^3 = U^3 \ln(C)$

(3) Differentiating with respect to time:

$$3(U^2 - e^B)^2 \left(2U\frac{dU}{dt} - e^B \frac{dB}{dt}\right) = 3U^2 \frac{dU}{dt} \ln(C) + U^3 \frac{\frac{dC}{dt}}{C}$$

(4) We know $B = \ln(2)$, $\frac{dB}{dt} = -2$, $C = e$, and $\frac{dC}{dt} = \frac{3e}{2}$.

We need to figure out what U is, and for this we use the equation for U:

$$\begin{aligned}(U^2 - e^B)^3 =& U^3 \ln(C)\\ (U^2 - e^{\ln(2)})^3 =& U^3 \ln(e)\\ (U^2 - 2)^3 =& U^3\\ U^2 - 2 =& U\\ U^2 - U - 2 =& 0\\ (U-2)(U+1) =& 0\end{aligned}$$

Which gives $U = 2$ or $U = -1$. However, we know that $U > 0$, so $U = 2$.

Now all that's left to do is to plug in everything:

$$
\begin{aligned}
3(U^2 - e^B)^2 \left(2U\frac{dU}{dt} - e^B\frac{dB}{dt}\right) =& 3U^2\frac{dU}{dt}\ln(C) + U^3\frac{\frac{dC}{dt}}{C} \\
3(2^2 - e^{\ln(2)})^2 \left(2(2)\frac{dU}{dt} - e^{\ln(2)}(-2)\right) =& 3(2)^2\frac{dU}{dt}\ln(e) + 2^3\frac{\frac{3e}{2}}{e} \\
3(4-2)^2\left(4\frac{dU}{dt} + 4\right) =& 12\frac{dU}{dt} + 8 \times \frac{3}{2} \\
48\frac{dU}{dt} + 48 =& 12\frac{dU}{dt} + 12 \\
36\frac{dU}{dt} =& -36 \\
\frac{dU}{dt} =& -1
\end{aligned}
$$

(5) Hence Peyam is getting **sadder** now, by 1 util per day.